Exam. Code : 211001

Subject Code : 3834

M.Sc. Mathematics 1st Semester

REAL ANALYSIS—I

Paper : MATH-551

Time Allowed—3 Hours] [Maximum Marks—100

Note :— (1) Attempt **five** questions in all, selecting at least **one** question from each part.

(2) Include the necessary results, depending upon the credit of the question.

(3) In the following, X denotes a metric space with metric d.

PART—I

1. (a) Prove that finite Cartesian product of countable sets is countable. 7

(b) Prove that open balls/neighborhoods in X are open subsets of X. 6

(c) Let $E \subset X$ and x be a limit point of E. Prove that for every $\in > 0$, the set $B(x ; \in) \cap E$ is infinite. 7

2. (a) Prove that every closed and bounded subset of R is compact. Is it true for arbitrary metric spaces ? 8

(b) Prove that every infinite compact metric space has a limit point. 6

(c) Prove that there are uncountably many irrational numbers inside the Cantor set. 6

PART—II

3. (a) Prove that [0, 1] is a connected subset of reals. 6

(b) Let A and B be separated subsets of X such that $A \cup B = X$. Prove that both A and B are open as well as closed in X. 6

(c) Prove that the sum as well as the product of two functions of bounded variation are functions of bounded variation. 8

4. (a) Does there exist a sequence which has uncountably many convergent subsequences ? Justify your answer. 6

(b) Let $\{x_n\}$ be a Cauchy sequence in X, containing a convergent subsequence. Prove that $\{x_n\}$ is a convergent sequence. 7

(c) State and prove the nested interval property of reals. 7

PART—III

5. (a) State and prove the Banach contraction principle. Also show that the completeness of the domain is not redundant. 8

(b) Prove that the set of all irrational numbers is not a countable union of closed subsets of reals. 6

(c) Prove that continuous image of compact sets is compact. 6

6. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be a monotone function discontinuous at some $d \in \mathbb{R}$. Prove that d is a jump discontinuity of f. 10

(b) Prove that the composition of two uniformly continuous functions, if possible, is uniformly continuous. 10

PART—IV

7. (a) State and prove the well known inequality consisting of the upper and lower Riemann integral of a bounded function. 10

(b) Prove that every continuous function has an antiderivative. 10

8. (a) Prove that every monotone real function on [0, 1] is Riemann-Stieltje integrable. 10

(b) State and prove the well known *integration by parts* formula for the Riemann integral. 10